Slicing a black hole by falling observers

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I investigate a family of spatial slices of Schwarzschild-Droste spacetime which are orthogonal to the worldlines of freely-falling observers. These observers fill all of spacetime and move radially at the same rate, specifically with the same Killing vector invariant e dubbed energy per mass at infinity. It is convenient to use new coordinates which replace Schwarzschild-t with the proper time of the observers; these further extend a generalisation of Gullstrand-Painleve coordinates made by Martel & Poisson (2001) and others. In contrast, the usual slices of Schwarzschild $t = \text{const}$ are orthogonal to the static Killing vector field, and hence are the 3-spaces determined by static observers.

The new slicing yields a different simultaneity convention (which by Frobenius' theorem is consistently defined), in which the time at infinity for an object to cross the horizon is only finite. Likewise the 3-spaces in this splitting are not the static slices, hence have different properties. The radial proper distance becomes $dr/|e|$, which reduces to the familiar quantity $(1 - 2M/r)^{-1/2}dr$ as a special case. Likewise, the 3-volume inside the horizon is simply $1/|e|$ times the Euclidean volume. The embedding diagram is a cone, in contrast to Flamm's funnel derived from the static slices. In the new coordinates time and space don't swap roles at the horizon. In future work I hope this alternate slicing could be relevant to QFT on curved spacetime, including the vacuum state or Hawking radiation, but for the time being I invite informed audience speculation on this possibility.

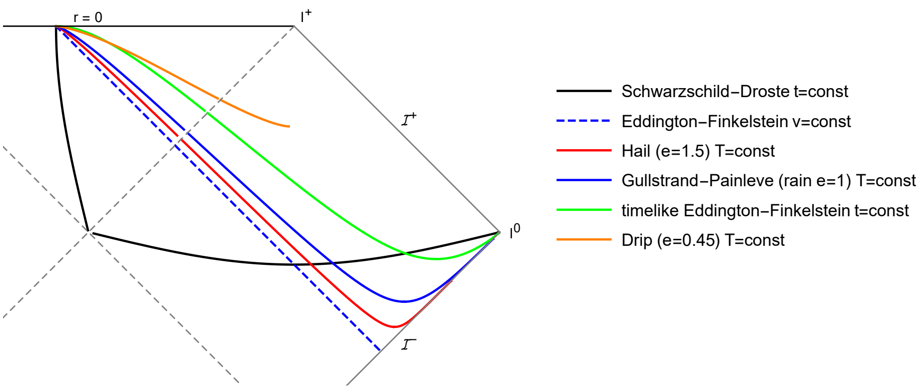

Figure 1: Simultaneity hypersurfaces in a Penrose diagram. See particularly the "rain" ($e = 1$), "hail" ($e > 1$) and "drip" ($0 < e < 1$) cases; $e < 0$ is not shown.